

Analysis of the Effects of the COVID-19 Crisis on the Persistence and Asymmetry of Volatility in the Paris Stock Market

Analyse des Effets de la Crise de la COVID-19 sur la Persistance et l'Asymétrie de la Volatilité du Marché Boursier de Paris

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Abstract

The purpose of this article is to analyze the impact of the COVID-19 crisis on the phenomenon of persistence and asymmetry in the volatility of the Paris stock market. To do this, we have split our series into two periods before (from 01/03/2017 to 10/23/2021) and during (from 01/24/2020 to 10/21/2021) the COVID-19 crisis based on the appearance of the first case of COVID-19 in France (January 24, 2020). Subsequently, we applied a GARCH model to capture the phenomenon of volatility persistence and the EGARCH and GJR-GARCH models for asymmetric effects. The conclusions of this work led firstly to the confirmation of the increase in the persistence of the volatility of the CAC All-Share and CAC 40 indices and secondly to the appearance of an asymmetry effect in the behavior of volatility. These results imply that unlikely events outside the financial markets produce turbulence and uncertainties that drive the level of volatility to very high peaks following fluctuations in the valuation of financial assets listed on the stock exchange.

Keywords : Conditional volatility ; GARCH modeling ; Volatility persistence ; Asymmetry effect of negative shocks ; COVID-19 crisis.

Résumé

L'objet de cet article est d'analyser l'impact de la crise de la COVID-19 sur le phénomène de persistance et d'asymétrie de la volatilité du marché boursier de Paris. Pour se faire, nous avons scindé notre série en deux périodes avant (du 03/01/2017 au 23/10/2021) et pendant (du 24/01/2020 au 21/10/2021) la crise de la COVID-19 sur la base d'apparition du premier cas de COVID-19 en France (24 Janvier 2020). Par la suite, nous avons appliqué un modèle GARCH pour capter le phénomène de persistance de la volatilité et les modèles EGARCH et GJR-GARCH pour les effets asymétriques. Les conclusions de ce travail ont abouti dans un premier temps à la confirmation de l'augmentation de la persistance de la volatilité des indices CAC All-Share et CAC 40 et dans un second temps, à l'apparition d'un effet d'asymétrie dans le comportement de la volatilité. Ces résultats impliquent que les événements improbables et extérieurs aux marchés financiers produisent des turbulences et des incertitudes qui conduisent le niveau de la volatilité à des pics très élevés suite aux fluctuations des valorisations des actifs financiers cotés à la bourse.

Mots clés : Volatilité conditionnelle ; Modélisation GARCH ; Persistance de la volatilité ; Effet d'asymétrie des chocs négatifs ; Crise de la COVID-19.

Introduction

Volatility, the consequence of price fluctuations, is essential for stock market investors. It is the raw material of market activities and provides a counterparty function to cover the risks of the real economy. So trying to reduce it to nothing would be useless. On the one hand, the market portfolio is always characterized by a systematic and specific risk whose magnitude depends on macro and micro economic expectations, sometimes taking into account geopolitical factors. On the other hand, without volatility there is no price movement and therefore no possible gain. The fundamental issue is that the market must find the right balance, in other words, we cannot accept either excessive upward or downward price volatility, or very low volatility in order to preserve the attractiveness of stock markets. However, several empirical studies have demonstrated certain stylized facts that characterize the behavior of volatility: excessive volatility of the markets that disrupt the formation of the price of financial assets (Shiller 1981), the non-stability or absence of the homoscedasticity of volatility over time and its character that is increasingly predictable, the leverage or asymmetry effect, and finally the short-term and long-term persistence of volatility.

ARCH or GARCH type models that take these properties into consideration are the conditional heteroscedasticity models. The modelling of the so-called conditional volatility from these models allows both the analysis of the persistence and the forecasting of the volatility. The implications of this modelling are very useful in terms of asset allocation for portfolio managers because the assumptions of modern portfolio management theory are no longer verified, particularly the normality of the distribution of return series.

The ongoing instability of volatility has certainly been amplified by the COVID-19 pandemic. This unprecedented crisis, both in its nature and in the magnitude of its consequences, affected a large part of the world and undermined the global economy. Capital markets experienced very high volatility and significant declines in financial asset prices, necessitating the adoption of appropriate measures by public authorities and financial market regulators.

We will try to find out whether or not the COVID-19 crisis has altered the persistence and asymmetry of volatility in the Paris stock market indices. We choose this market because France is the fifth largest country in the world and the eighth most affected by the COVID-19 pandemic. In addition, it represents an important weight for the European Union due to its economic dynamism.

The purpose of this paper is to analyze the behavior of the volatility of the Paris stock market, before and during the COVID-19 pandemic by admitting a comparative approach between the

main indices of the Paris market : CAC All-Share and CAC 40. In other words, this analysis will allow us to see to what extent the COVID-19 crisis produced a short- or long-term volatility shock and to amplify the asymmetry effect by conducting a comparative analysis between the two indices CAC All-Share and CAC 40.

The central problem of this study is : How has the persistence of volatility been impacted by the COVID-19 crisis ? And, did this crisis create or accentuate asymmetry ?

After the introduction, this study presents a methodology and a review of the literature on volatility models and the asymmetry effect (1), followed by an analysis and modeling of the volatility of the Paris stock market (2) which includes the results and discussions.

1. Methodology and a review of the literature on volatility models and the asymmetry effect

1.1. The ARCH model (Engel, 1982)

The ARCH (q) process is the basic model of ARCH processes proposed by Engel (1982) in a study of the variance of inflation in Britain. The ARCH (q) model is based on a quadratic parameterization of the conditional variance. Appears σ_t^2 as a linear function of the q past values of the squared innovation process.

An ARCH (q) process is given by :

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 = \alpha_0 + \alpha(L) \varepsilon_t^2 \quad (1)$$

Where : $\alpha_0 > 0$ and $\alpha_i \geq 0 \forall i$.

The constraints on the coefficients guarantee the positivity of the conditional variance. Moreover, we can show that this variance is finite if $\sum_{i=1}^q \alpha_i < 1$.

The ARCH (q) model allows for volatility clustering, in other words the fact that strong (respectively weak) price changes are followed by other strong (respectively weak) price changes but whose sign is not predictable.

Bollerslev (1986) generalized Engel's original model by establishing the GARCH (p,q) model (*Generalized ARCH*). This extension consists in introducing lagged values of the variance in his equation and is, therefore, similar to the extension of AR processes to ARMA. This allows a more parsimonious description of the lag structure.

1.2. The GARCH model (Bollerslev, 1986)

A GARCH process (p, q) is defined by :

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 = \alpha_0 + \alpha(L) \varepsilon_t^2 + \beta(L) \sigma_t^2 \quad (2)$$

Where : $\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0, \forall i, \forall j$.

Again, the constraints on the coefficients ensure the positivity of the conditional variance.

Of course, when $p = 0$, the GARCH (p, q) process reduces to the ARCH (q) process.

An important property of the GARCH process is that it can be interpreted as an ARMA process on the innovation square.

Let's put it this way :

$$u_t = \varepsilon_t^2 - \sigma_t^2 \quad (3)$$

Let's replace σ_t^2 by $(\varepsilon_t^2 - u_t)$ in equation (2) :

$$\varepsilon_t^2 - u_t = \alpha_0 + \alpha(L) \varepsilon_t^2 + \beta(L) [\varepsilon_t^2 - u_t] \quad (4)$$

$$[I - \alpha(L) - \beta(L)] \varepsilon_t^2 = \alpha_0 + [I - \beta(L)] u_t \quad (5)$$

Thus, equation (5) is interpreted as an ARMA (max (p, q), p) on the series of squared residuals.

This new writing allows us to easily write the conditions of weak stationarity. These are based on the existence of a variance that is $V(\varepsilon_t) = E(\varepsilon_t^2)$ asymptotically independent of time.

A necessary and sufficient condition for stationarity of the GARCH process (p, q) is (see Bollerslev (1986)) :

$$\alpha(1) + \beta(1) = \sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1 \quad (6)$$

We simply translate here the fact that if the roots of the polynomial :

$$1 - \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) L^i \quad (7)$$

Are outside the unit disk, then the series $E(\varepsilon_t^2)$ converges and the process becomes asymptotically stationary at order two.

The most widespread specification is the GARCH (1,1) which allows a fairly general representation of conditional volatility processes. In its simplest form, the GARCH (1,1) is written as follows with $(\alpha_0 = \omega)$:

$$GARCH(1,1): \sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (8)$$

ω : The unconditional variance.

α_1 : Is a parameter that governs the impact of the past stock.

β_1 : Is a parameter that can be interpreted as the speed of return to minimum volatility ω .

We note that $\omega = \gamma V_L$, the model is written as follows :

$$GARCH(1,1): \sigma_t^2 = \gamma V_L + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (9)$$

γ , α and β represent the weights assigned to V_L (unconditional volatility or long-term volatility), ε_{t-1}^2 et σ_{t-1}^2 . In addition, the sum of the weights is equal to 1 ($\gamma + \alpha + \beta = 1$).

This formulation is generally used to estimate the parameters ω , α and β . From this estimate we can calculate γ which is $\gamma = 1 - \alpha - \beta$ and the long term variance $V_L = \frac{\omega}{\gamma}$. The stability of the GARCH process is obtained by imposing $\alpha + \beta < 1$. If not, the weight applied to the long term variance will be negative.

The GARCH model is a solution capable of reproducing two stylized facts of financial series namely :

- First, clustering or grouping of return volatility by cluster as demonstrated by Mandelbrot. Returns small in size are followed by returns small in size and returns large in size are followed by returns large in size with quieter periods.
- Second, the non-normality of the distribution of returns. This distribution is often leptokurtic, meaning that the probability of extreme positive or negative returns is greater than that of a normal distribution. Financial time series generally have thick tails that are referred to as excess kurtosis. Thus, the kurtosis coefficient is generally greater than the kurtosis of a Gaussian random variable.

Generally the GARCH (1,1) is commonly used in the modelling of financial series, and implies a persistence of respectively low and infinite volatility shocks. The persistence in the GARCH model is measured by the sum of the coefficients ($\alpha + \beta$). The more this sum tends towards 1, without being higher ($\alpha + \beta < 1$) to guarantee the stability of the model, the more persistent the volatility is.

The GARCH model allows for the persistence of volatility that can be predicted with a return to unconditional volatility (V_L). However, it is unable to reproduce asymmetry effects. In order to fill this gap, we will use asymmetric GARCH models (EGARCH, GJR-GARCH and TGARCH).

Most empirical studies (Engle and Bollerslev 1986, French, Schwert and Starnbaugh 1987, Chou 1988, Baillie and DeGennaro 1990, Campbell and Hentschel 1992) accept the hypothesis of strong persistence in high-frequency volatility. On the other hand, the persistence tends to decrease significantly as the frequency of the data decreases, which provides an indication of

the stationarity of the conditional variance (Poon and Taylor, 1992). As this study was conducted with daily data, volatility persistence phenomena can therefore be observed.

1.3. Conditional volatility and asymmetry

In their initial version, the ARCH and GARCH models do not allow for the asymmetry assumption. To correct this shortcoming, more recent work has proposed extensions of this approach to take account of these effects. Several formulations have been proposed (Nelson 1991, Engle and Ng 1993, Glosten, Jagannathan, and Runkle 1993, Zakoian 1994, and Hentschel 1995). Asymmetry leverage distinguishes between the effect of negative past values and the effect of positive past values on price volatility or returns. Price declines tend to increase volatility by more than a price increase of the same magnitude.

The GARCH model is a response of future volatility to price changes determined only by the magnitude of the price change and not by its sign. In contrast, asymmetric GARCH models extend these specifications by incorporating asymmetry in the response of volatility to price changes. Three specifications are particularly noteworthy.

1.3.1. The EGARCH model (Nelson, 1991)

The EGARCH model differs from the usual ARCH models in that it rejects the symmetry assumption associated with the quadratic specification of the variance.

This is a log-linear model presented by Nelson (1991) in a study of the returns on financial assets. The specification concerns the logarithm of the conditional variance and thus avoids the positivity constraints on the coefficients α_i and β_i .

The EGARCH (p, q) model is written as follows :

$$\ln \sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i (\varepsilon_{t-i} + \gamma [|z_{t-i}| - E|z_{t-i}|]) + \sum_{j=1}^p \beta_j \ln \sigma_{t-j}^2 \quad (10)$$

$$\text{Where : } z_{t-i} = \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \quad (11)$$

According to Nelson (1991), the positivity constraints on the coefficients of the GARCH models (p, q) were often violated in practice, and also eliminated any possibility of cyclical behaviour. This criticism is no longer valid in the case of exponential GARCH models since the coefficients can be positive or negative.

Indeed, either :

$$g(z_t) = \emptyset + \gamma[|z_t| - E|z_t|] \quad (12)$$

- If $z_t > 0$, then $g(z_t)$ is a function of slope $\gamma + \emptyset$.
- If $z_t < 0$, the slope becomes $\gamma - \emptyset$.

Thus, the conditional variance responds asymmetrically to the sign of the innovations z_{t-i} . The sign and magnitude effects are respectively taken into account by the coefficients $\emptyset$ and γ .

As Nelson (1991) points out, the choice of “standardized” variables z_{t-i} rather than ε_{t-i} makes it possible to obtain conditions of weak stationarity relating only to the polynomial $\beta(L)$.

Finally, our specific EGARCH model which is the EGARCH (1,1) with $\alpha_0 = w$ is written as follows :

$$\ln \sigma_t^2 = w + \alpha_1 \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \ln \sigma_{t-1}^2 \quad (13)$$

This implies that the leverage effect is exponential rather than quadratic, and ensures that the conditional variance predictions are not negative. The presence of leverage effects can be assessed under the assumption that $\gamma = 0$ or not. The impact is asymmetric if $\gamma \neq 0$ and when the effect is symmetric, in other words, positive or negative price changes produce the same magnitudes of volatility.

The conditional variance σ_t^2 shows a sign effect, corresponding to $\gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$ an amplitude effect measured by $\alpha_1 \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right|$.

The coefficient $\alpha_1 + \gamma$ measures the total effect of a positive shock on volatility and for a negative shock, the total impact corresponds to the coefficient $\alpha_1 - \gamma$.

Using the EGARCH model, Black (1976) finds that volatility in the stock market tends to increase after negative returns and tends to decrease after positive returns. The EGARCH model exploits this empirical regularity by putting the conditional variance as a function of the size and sign of lagged residuals which is different from the GARCH (p, q) model.

1.3.2. The GJR-GARCH model (Glosten Jagannathann and Runkel, 1993)

The GJR-GARCH model is defined by :

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \gamma_i I_{t-i} \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad \text{with} \quad I_{t-i} = \begin{cases} 1 & \text{si } \varepsilon_{t-i} < 0 \\ 0 & \text{si } \varepsilon_{t-i} \geq 0 \end{cases} \quad (14)$$

To ensure the positivity of the volatility it is necessary that: $\omega > 0$; $\alpha_i \geq 0$; $\alpha_i + \gamma \geq 0$ and $\beta_j \geq 0$

Our specific model which is the GJR-GARCH (1,1) is written as follows : $\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 I_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ with $I_{t-i} = \begin{cases} 1 & \text{si } \varepsilon_{t-1} < 0 \\ 0 & \text{si } \varepsilon_{t-1} \geq 0 \end{cases} \quad (15)$

and $\omega > 0$; $\alpha_1 \geq 0$; $\alpha_1 + \gamma \geq 0$ and $\beta_1 \geq 0$

The model is stable provided that : $\alpha_1 + \beta_1 + \frac{\gamma_1}{2} < 1$ is verified¹. (16)

In particular, the coefficient γ_1 allows us to measure the additional effect of a negative shock on volatility, given that the total impact corresponds to the sum of $\alpha_1 + \gamma_1$. In the case of a positive shock, the total impact corresponds to the coefficient α_1 .

1.3.3. The TGARCH model (Threshold GARCH, Zakoian 1994)

In this threshold GARCH formulation, introduced by Zakoian (1994), the quadratic form of equation (3) is replaced by a piecewise linear function. Each of the segments is associated with shocks of the same “nature”, which makes it possible to obtain different volatility functions depending on the sign and the values of the shocks.

A TGARCH (p,q) process is written :

$$\sigma_t = \alpha_0 + \sum_{i=1}^q (\alpha_i^+ \varepsilon_{t-i}^+ - \alpha_i^- \varepsilon_{t-i}^-) + \sum_{j=1}^p \beta_j \sigma_{t-j} \quad \text{with} \quad \alpha_0 > 0 ; \alpha_i^+ \geq 0 ; \alpha_i^- \geq 0 ; \beta_j \geq 0. \quad (17)$$

$$\sigma_t = \alpha_0 + \alpha^+(L) \varepsilon_t^+ - \alpha^-(L) \varepsilon_t^- + \beta(L) \sigma_t \quad \text{where} \quad \begin{cases} \varepsilon_t^+ = \max(\varepsilon_t, 0) \\ \varepsilon_t^- = \min(\varepsilon_t, 0) \end{cases} \quad (18)$$

Insofar as the specification does not concern a square but the conditional standard deviation, it is possible to remove the positivity constraints on the coefficients. The removal of these constraints makes it possible to take into account the asymmetry phenomena described above

¹ See Hentschel, 1995.

concerning volatility. The effect of a shock ε_{t-i} on the conditional variance will thus depend on both the amplitude and the sign of this shock.

Our specific model which is TGARCH (1,1) is written as follows :

$$\sigma_t = \alpha_0 + \alpha_1^+ \varepsilon_{t-1}^+ - \alpha_1^- \varepsilon_{t-1}^- + \beta_1 \sigma_{t-1} \quad (19)$$

with $\alpha_0 > 0$; $\alpha_1^+ \geq 0$; $\alpha_1^- \geq 0$; $\beta_1 \geq 0$ to ensure the positivity of the volatility.

The TGARCH model allows the same interpretation of the model's coefficients in the case of a negative or positive shock. The fundamental difference between the two models lies in its formulation. The GJR-GARCH is an expression of the conditional variance and the TGARCH is an expression of the conditional volatility. This is why in empirical studies, the GJR-GARCH model is preferred to the TGARCH specification. In contrast, the EGARCH is an expression of the logarithm of the conditional volatility.

2. Analysis and modeling of the volatility of the Paris stock market

2.1. Data

For the purpose of this analysis, we use daily data from the CAC All-Share and the CAC 40. Our data covers the period from 02 January 2017 to 21 October 2021, which gives a number of 1229 observations to be analyzed. The stock price dynamics can be found in the appendices (see figures 1 and 2 in the appendices). The data are available on the following official website : www.investing.com. We then compute the stock returns in continuous components (Campbell, Lo and Mackinlay, 1996).

The calculation method is as follows :

$$R_t = \ln \left(\frac{p_t}{p_{t-1}} \right) \times 100 \quad (20)$$

With p_t , value of the index at date t and p_{t-1} , value of the index at period $t - 1$. See figures 3 and 4 (in the appendices) for the evolution of the returns. Finally, the analysis of the impact of the COVID-19 crisis on the persistence and asymmetry of volatility will be carried out over two periods : before (from 03/01/2017 to 23/10/2021) and during COVID-19 (from 24/01/2020 to 21/10/2021). The breakdown is based on the announcement date of the first case of COVID-19 in France (24 January 2020).

2.2. Analysis of the characteristics of the daily returns of the CAC All-Share and CAC 40

The results of the descriptive statistics of the daily returns of the two indices spanning between 02/01/2017 and 21/10/2021 are summarized in the following table :

Table 1 : Descriptive statistics of daily yields.

	CAC All	CAC 40
Number of observations	1228	1228
Mean	0,0331	0,0256
Median	0,0806	0,0777
Maximum	7,9406	8,0560
Minimum	-12,4211	-13,0983
Std-deviation	1,1199	1,1806
Skewness	-1,4240	-1,3900
Kurtosis	22,8028	22,7947
Jarque-Bera	20480,16	20444,22
Probability	0,0000	0,0000

Source : Author, based on daily returns of CAC All and CAC 40.

For normally distributed series, the skewness and kurtosis coefficients are equal to 0 and 3, respectively. Both indices have Skewness different from 0 and negative. This implies that volatility is higher after a decline than after an increase in returns and the density of the distribution is concentrated to the left. This implies that the returns react more to a negative shock than to a positive one. The kurtosis coefficients are greater than 3. We are thus in the presence of leptokurtic distributions that have thick tails compared to the extremities of a normal distribution (see figures 5 and 6 in the appendices for representations of the densities). These observed characteristics finally give a distribution different from the normal distribution, which is confirmed by the Jarque-Bera statistic (0,0000 less than 0,05).

2.3. Empirical observation of the realized volatility of the Paris stock market

The empirical analysis of the volatility of the main indices can be approached according to two horizons (long and short term) :

- A long-term horizon by calculating the volatility of logarithmic returns over a 250-day period (annual volatility²).
- A short-term horizon by calculating the volatility over a period of 20 days (monthly volatility³).

$$\bar{R} = \frac{1}{n-1} \sum_{t=1}^n R_t \quad (21)$$

with $\bar{R}$ which represents the daily average logarithmic return, and thus the standard deviation

$$j \text{ is equal to : } \sigma_j(R) = \sqrt{\frac{1}{n-1} \sum_{t=1}^n (R_t - \bar{R})^2}. \quad (22)$$

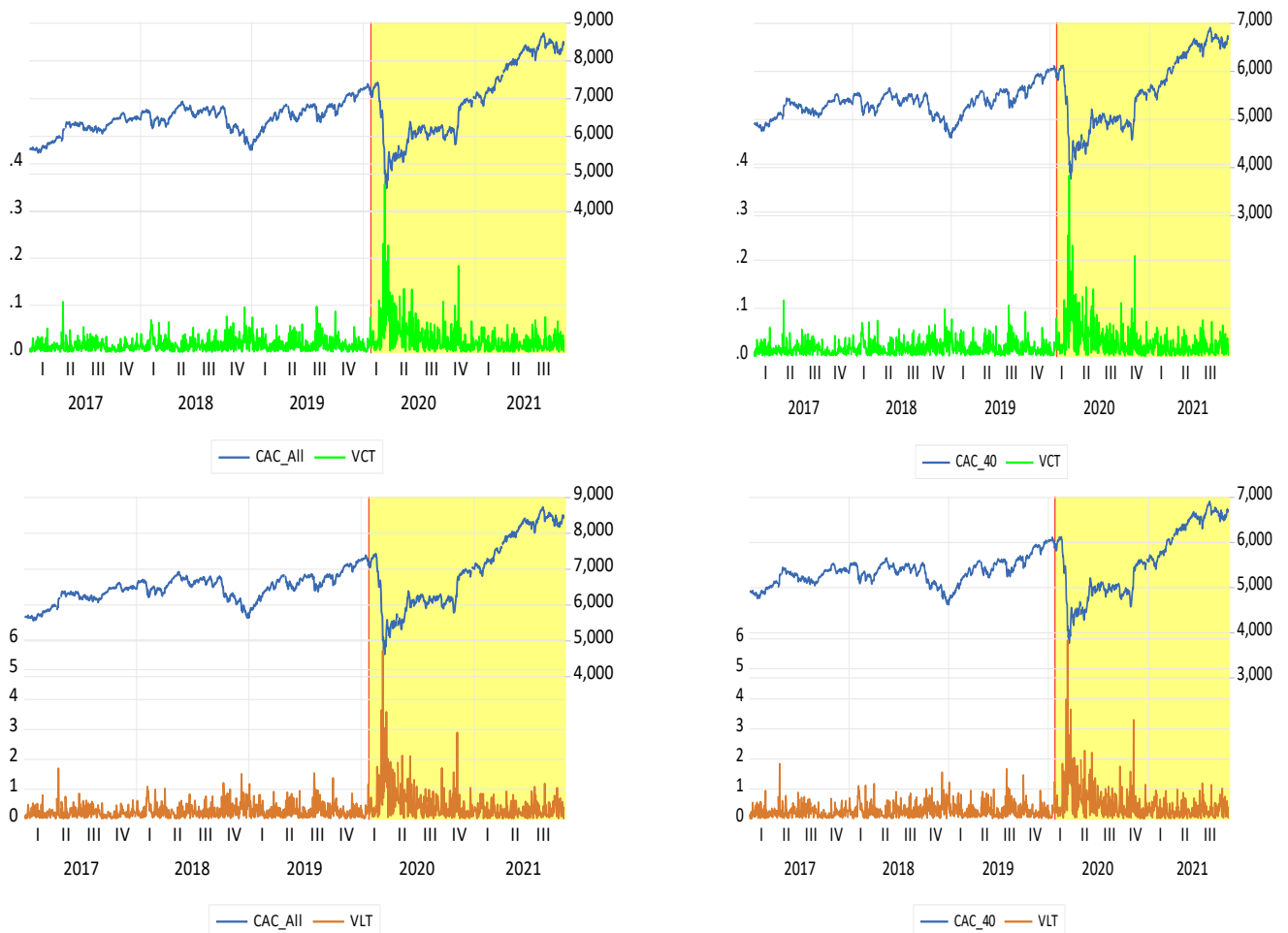
From the daily volatility, we can calculate the annual volatility :

$$\sigma(R)_{Annual} = \sqrt{250} * \sigma_j(R). \quad (23)$$

² The average number of trading session per year is 250.

³ The number of sessions per month is 20, or 5 sessions per week.

Graph 1 : Changes in the historical short and long-term volatility of CAC All and CAC 40 indices between 02/01/2017 and 21/10/2021.



Source : Author, our calculations.

A graphical examination of historical volatilities reveals a sharp explosion in short- and long-term volatility for both indices since the beginning of the COVID-19 crisis period (yellow area) marked by the appearance of the first confirmed case in France on 24 January 2020. Afterwards, following the measures taken by the French and European authorities, things started to return to normal.

The strong fluctuations in volatilities with both upward and downward amplitudes confirm the strong dispersion or temporal instability of volatilities. This instability has been strongly accentuated since the beginning of the COVID-19 pandemic. Moreover, periods of low volatility follow periods of high volatility with a peak since the installation of COVID-19. This confirms the persistence of volatility with an asymmetry effect since these graphs show an inverse relationship between prices and volatilities.

Indeed, following the COVID-19 crisis, the collapse of stock prices caused investors' portfolios ("junk") to lose value, which led to market panic and massive selling (sell-off). This massive sell-off leads to a spectacular increase in volatility. This volatility measures the risk of loss for investors in the market. We deduce that the COVID-19 crisis produced a leverage or asymmetry effect to be confirmed. The instability of volatility is a major handicap for portfolio risk management using the mean-variance approach⁴.

Table 2 : Maximum, Minimum and Median of the two indices before and during the COVID-19 crisis.

	15 CRISI:				
	Indexes	CAC ALL		CAC 40	
	Standar deviation(σ)	σ_{CT}	σ_{LT}	σ_{CT}	σ_{LT}
Before the COVID-19 (03/01/2017 - 23/10/2021)	Maximum	10,601 %	167,6166 %	11,5138 %	182,0496 %
	Minimum	7,85 ^e -06	1,24 ^e -04	8,48 ^e -06	1,34 ^e -04
	Median	1,1278 %	17,8325 %	1,2167 %	19,2376 %
During the COVID-19 (24/01/2020 - 21/10/2021)	Maximum	35,54 %	561,9368 %	37,4512 %	592,1551%
	Minimum	8,41 ^e -05	1,33 ^e -03	3,03 ^e -05	4,79 ^e -04
	Median	1,7222 %	27,2312 %	1,892 %	29,9152 %

Source : Author, our calculations.

Table 2 clearly demonstrates the impact of the COVID-19 crisis in the increase in realized volatility. The level of the Maximum and Median values have been exploded compared to the period before the COVID-19 crisis.

2.4. Volatility modelling : an application to the Paris stock market

This modeling should allow us to answer two questions. How has the persistence of volatility been impacted by the COVID-19 crisis ? And, did this crisis create or accentuate asymmetry ?

2.4.1. Analysis of volatility persistence for the GARCH model

To analyze the impact of the COVID-19 crisis, we model in two periods :

An initial pre-crisis modeling of COVID-19 based on data from 03/01/2017 to 23/10/2021.

A second modelling during the COVID-19 crisis since we are still living with this pandemic : 24/01/2020 - 21/10/2021. The breakdown is based on the date of announcement of the first case of COVID-19 in France (24 January 2020).

Before proceeding to the different estimations, we will first test the ARCH effect in the series of the CAC All and CAC 40 returns.

⁴ The mean-variance approach is the basic model for portfolio allocation and was developed by Markowitz.

Table 3 : ARCH test results for CAC All returns.

RESID^2	Coeff.	Std.Error	t.Stat	Prob.
C	1,1227	0,1700	6,6039	0,0000
RESID^2(-1)	0,1053	0,0284	3,7073	0,0002
F-statistic	13,7443	-	-	0,0002
Obs*R-squared	13,6139	-	-	0,0002

Source : Author, our estimates.

Table 4 : ARCH test results for CAC 40 returns.

RESID^2	Coeff.	Std.Error	t.Stat	Prob.
C	1,2528	0,1891	6,6219	0,0000
RESID^2(-1)	0,1019	0,0284	3,5840	0,0004
F-statistic	12,8451	-	-	0,0004
Obs*R-squared	12,7325	-	-	0,0004

Source : Author, our estimates.

The results of the ARCH test of the CAC All and the CAC 40 show us that the autoregressive coefficient associated with the lagged residuals (RESID^2(-1)) is positive and significantly different from zero. Thus, there is a presence of conditional heteroscedasticity in the return series. In addition, we find that the probability associated with the test statistic TR^2 (Obs*R-squared) is zero : we therefore reject the null hypothesis of homoscedasticity in favour of the alternative of conditional heteroscedasticity. In order to take into account this ARCH effect, our objective is now to estimate the variance equation, together with the mean equation. We will use the GARCH model for this purpose. The results of the estimations of the GARCH (1,1) model using the GED (Generalized Error Distribution) are given in Table 5 below.

Table 5 : Results of GARCH (1,1) modelling with GED for the CAC All and CAC 40 indices.

Indexes	CAC All		CAC 40	
	Before	During	Before	During
Periods				
ω	0,0451** (0,0111)	0,0607*** (0,0807)	0,0624** (0,0105)	0,0606*** (0,0873)
α_1	0,1585* (0,0001)	0,1557* (0,0026)	0,1587* (0,0002)	0,1472* (0,0015)
β_1	0,7699* (0,0000)	0,8156* (0,0000)	0,7505* (0,0000)	0,8267* (0,0000)
Persistence Coefficient $(\alpha_1 + \beta_1)$	0,9284	0,9713	0,9092	0,9739
$\gamma = 1 - (\alpha_1 + \beta_1)$	0,0716	0,0287	0,0908	0,0261
Annualized volatility $\sqrt{V_L} * \sqrt{250}$	1254,8787 %	2299,4469 %	1310,7485 %	2409,2732 %

Source : Author, our estimates. (*), (**), (***) indicator of significance respectively at 1 %, 5% et 10%.
Values in parentheses are p-values.

The persistence phenomenon in the GARCH (1,1) model is measured by the coefficient $(\alpha_1 + \beta_1)$. The results show us that the persistence of the volatility of the CAC All is higher than that of the CAC 40 index. Regarding the impact of the COVID-19 crisis, the overall volatility of the two indices has increased significantly and even doubled. However, the increase in volatility of the CAC 40 is higher than that of the CAC All during the COVID-19 crisis. This clearly shows the effect of the COVID-19 crisis on the increase in volatility of both indices. With a persistence coefficient around 90% for both indices, indicates that a return to normal by tending towards unconditional volatility requires time by taking into consideration the possible factors that allow this situation to persist.

2.5. Analysis of the impact of the COVID-19 crisis and the asymmetry effect

To verify the asymmetry effect on volatility, we will limit ourselves to the use of the EGARCH and GJR-GARCH models from the Eviews 12 software, distinguishing two periods : before and during the COVID-19 crisis.

2.5.1. EGARCH modelling

The results of the estimations of the EGARCH (1,1) model using the GED (Generalized Error Distribution) are summarized in the following table :

Table 6 : Results of the EGARCH (1,1) modelling with GED for the CAC All and CAC 40.

Indexes	CAC All		CAC 40	
Periods	Before	During	Before	During
w	-0,1918* (0,0002)	0,0355** (0,0310)	-0,1550* (0,0005)	0,0335** (0,0434)
α_1	0,1449* (0,0047)	-0,0682* (0,0033)	0,1158** (0,0112)	-0,0649* (0,0057)
γ	-0,2562* (0,0000)	-0,1488* (0,0000)	-0,2601* (0,0000)	-0,1551* (0,0000)
β_1	0,9006* (0,0000)	0,9949* (0,0000)	0,9001* (0,0000)	0,9941* (0,0000)

Source : Author, our estimates. (*), (**) indicator of significance respectively at 1 % and 5%.

Values in parentheses are p-values.

Examining the statistical significance of the different coefficients (γ) of the model clearly shows us that they are significantly different from zero ($\gamma \neq 0$) at the 1% level. This indicates that before and during the period of the COVID-19 crisis, the behavior of volatility shows a leverage effect from a statistical point of view for both indices. The presence of the asymmetric phenomenon shows that positive or negative price changes do not produce the same volatility magnitudes. The coefficient γ is negative ($\gamma < 0$) for both indices, indicating that bad news has a greater effect on stock volatility than good news before and since the crisis.

Furthermore, all coefficients of the variables in the variance equation are significantly different from zero before and during the COVID-19 crisis. Thus, there is indeed an asymmetry phenomenon, which could not be detected by the usual ARCH models.

Table 7 : Calculation of the total leverage on the $\ln \sigma_t^2$ CAC All and CAC 40 indices.

Indexes Periods	CAC All		CAC 40	
	Before	During	Before	During
If $\varepsilon_{t-1} > 0$, then the total effect is $(\alpha_1 + \gamma)$	-0,1113	-0,217	-0,1443	-0,22
If $\varepsilon_{t-1} < 0$, then the total effect is $(\alpha_1 - \gamma)$	0,4011	0,0806	0,3759	0,0902

Source : Author, our calculations.

We find that for good news ($\varepsilon_{t-1} > 0$), total leverage reduces the log of the conditional variance in all ($\ln \sigma_t^2$) periods. However, this reduction effect is very high during the COVID-19 crisis and less strong before.

On the bad news side ($\varepsilon_{t-1} < 0$), total leverage increases the logarithm of the conditional variance in all ($\ln \sigma_t^2$) periods. On the other hand, this increase diminishes during the COVID-19 crisis, which could be explained by the intervention of the French and European authorities to contain the pandemic.

2.5.2. GJR-GARCH modelling

Note that Eviews actually estimates the GJR-GARCH model when you select the GARCH/TARCH option and specify a threshold order. The original TARCH model works on the conditional standard deviation. However, the Eviews TARCH model uses the same specification as the GJR-GARCH model.

The results of the estimations of the GJR-GARCH (1,1) model using the GED (Generalized Error Distribution) are summarized in the following table :

Table 8 : Results of the GJR-GARCH (1,1) modelling with GED for the CAC All and CAC 40.

Indexes Periods	CAC All		CAC 40	
	Before	During	Before	During
ω	0,1647* (0,0001)	0,0108*** (0,0744)	0,0708* (0,0000)	0,0109*** (0,0985)
α_1	-0,0300* (0,0000)	-0,0231** (0,0109)	-0,0348* (0,0000)	-0,0185** (0,0316)
γ_1	0,4331* (0,0003)	0,1157* (0,0000)	0,3443* (0,0000)	0,1146* (0,0000)
β_1	0,5137* (0,0000)	0,9464* (0,0000)	0,7407* (0,0000)	0,9444* (0,0000)

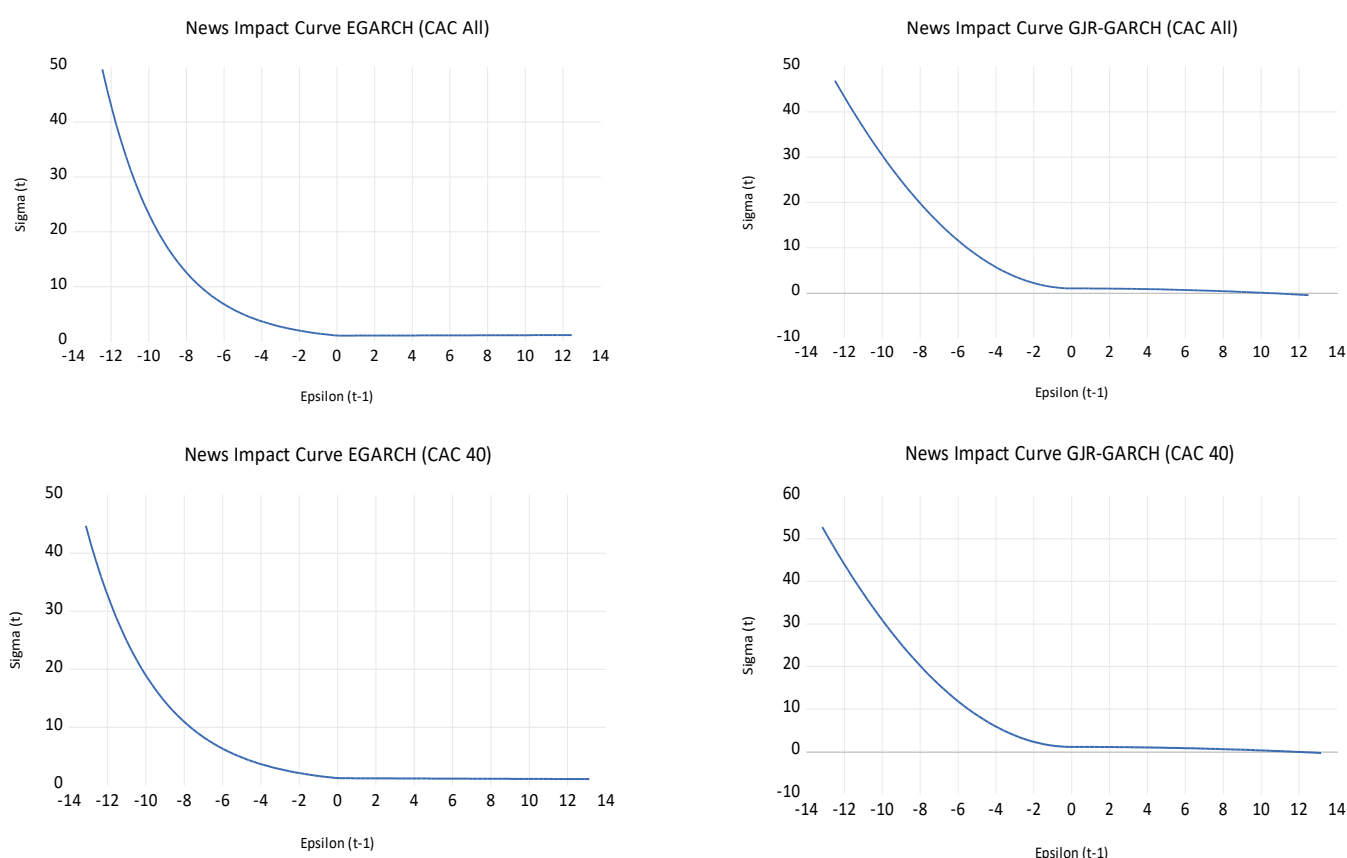
Source : Author, our estimates. (*), (**), (***) indicator of significance respectively at 1 %, 5% and 10 %. Value in parentheses are p-values.

For both indices, all the coefficients γ_1 are positive and significantly different from zero at the 1 % level, regardless for the period. This positive coefficient ($\gamma_1 > 0$) indicates that volatility will increase more strongly following a negative shock (bad news) or negative innovation than following a positive shock (good news). This highlights the relevance of this model in modelling the asymmetric behaviour of volatility.

However, we note a less pronounced asymmetry effect for both indices during the COVID-19 period, especially for large caps (CAC 40). This could be explained once again by the intervention of the French and European authorities to contain the pandemic.

The asymmetry can be visualized by the News Impact Curve for both the EGARCH and GJR-GARCH models :

Figure 7 : The information impact curve of the CAC All and CAC 40 indices by the EGARCH and GJR-GARCH models.



Source : Author, our estimates.

The choice of the perfect model depends on the comparison of the information criteria as summarized in the following table :

Table 9 : Comparison of criteria for selecting the optimal model.

Indexes Models	CAC All		CAC 40	
	EGARCH	GJR-GARCH	EGARCH	GJR-GARCH
Akaike info criterion	2,445870*	2,470519	2,567685*	2,592321
Schwarz criterion	2,475036*	2,499685	2,596851*	2,621487
Hannan-Quinn criter	2,456845*	2,481494	2,578660*	2,603295
Log likelihood	-1493,541*	-1508,664	-1568,275*	-1583,389
Adjusted R-squared	-0,001281*	-0,002665	-0,001257*	-0,002179

Source : Author, our estimates. (*) indicator for selecting the best model according to the criteria.

In the end, one should choose the model that minimizes the Akaike, Schwarz and Hannan-Quinn criteria and maximizes the Log likelihood and Adjusted R-squared criteria. It is clear that the EGARCH model is the best suited to describe and model the effects of asymmetry on conditional volatility for the CAC All and CAC 40 indices.

Conclusion

The analysis of the conditional volatility, before and during the COVID-19 crisis of the Paris stock market revealed some very relevant facts. On the one hand, the increase in the persistence of the volatility of the CAC All is higher than that of the CAC 40 index. In addition, the COVID-19 crisis has significantly increased and even doubled the overall volatility of both indices. However, the increase in the volatility of the CAC 40 is higher than that of the CAC All during this crisis.

On the other hand, there is a skewness effect confirmed by the EGARCH and GJR-GARCH models. The significantly non-zero skewness coefficient shows that positive or negative price changes do not produce the same magnitudes of volatility. The negative skewness coefficient of the EGARCH model for both indices indicates that bad news has a larger effect on stock volatility than good news before and since the crisis.

The positive skewness coefficient of the GJR-GARCH model for both indices also indicates that volatility will increase more strongly following a negative shock than following a positive shock. However, the skewness effect is less pronounced during the COVID-19 period, especially for the CAC 40, which could be explained by the intervention of French and European authorities to contain the pandemic.

Finally, the EGARCH model is best suited to describe and model the effects of skewness on the conditional volatility of our indices. The implications of the consequences of volatility persistence and skewness are of great use in portfolio allocation management to better understand asset risk and improve risk management.

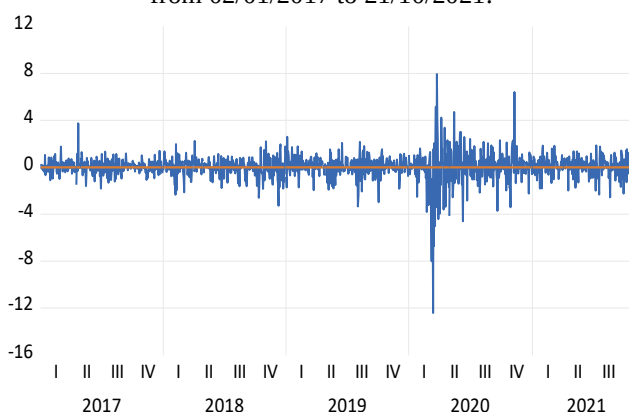
APPENDICES

Figure 1 : Dynamics of the CAC All index over the period from 02/01/2017 to 21/10/2021.



Source : Author, based on daily CAC All prices covering the period from 02/01/2017 to 21/10/2021.

Figure 3 : Changes in CAC All returns over the period from 02/01/2017 to 21/10/2021.



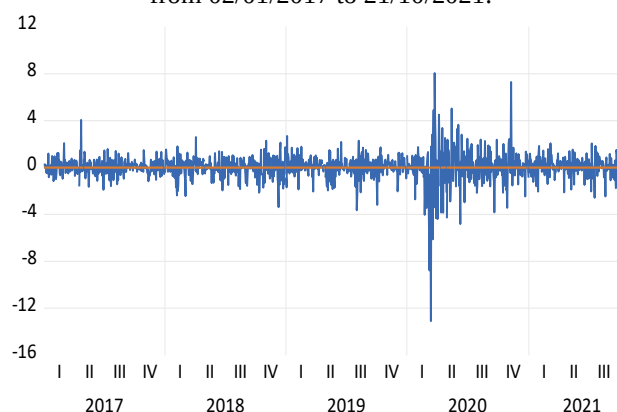
Source : Author, based on daily returns of the CAC All covering the period from 02/01/2017 to 21/10/2021.

Figure 2 : Dynamics of the CAC 40 index over the period from 02/01/2017 to 21/10/2021.



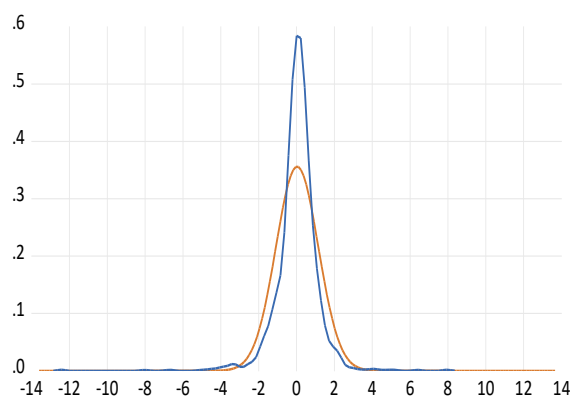
Source : Author, based on daily CAC 40 prices covering the period from 02/01/2017 to 21/10/2021.

Figure 4 : Changes in CAC 40 returns over the period from 02/01/2017 to 21/10/2021.



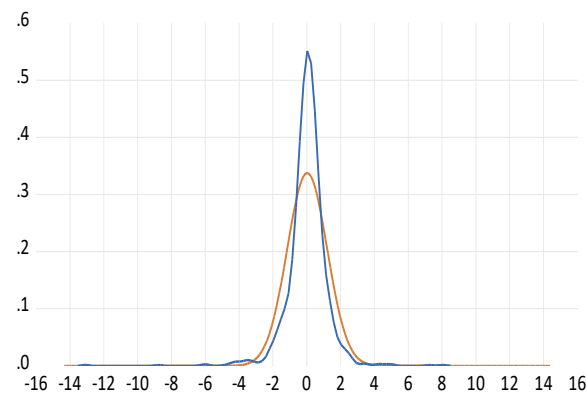
Source : Author, based on daily returns of the CAC 40 covering the period from 02/01/2017 to 21/10/2021.

Figure 5 : Representation of the density function of the distribution of daily returns of the CAC All (in blue) and a normal distribution (in red) over the period from 02/01/2017 to 21/10/2021.



Source : Author, based on daily returns of the CAC All Covering the period from 02/01/2017 to 21/10/2021.

Figure 6 : Representation of the density function of the distribution of daily returns of the CAC 40 (in blue) and a normal distribution (in red) over the period from 02/01/2017 to 21/10/2021.



Source : Author, based on daily returns of the CAC 40 Covering the period from 02/01/2017 to 21/10/2021.

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